Date:

Lesson Title: 3.1 Recursive Sequences

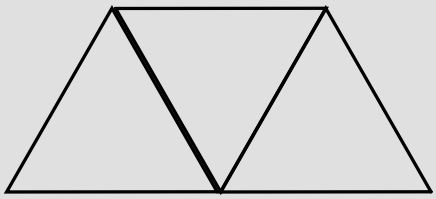
Objective: To analyze geometric patterns and find missing values using numerical sequences.

IN:

What is the perimeter of the figure below?

What units did you use?

What would be the perimeter if the toothpicks were each equal to 3 cm?



Hmmmm...How high is the Empire State Building in New York?

How high is the 80th floor?

Write your guesses down in your interactive notebook.

We are going to answer this question using a RECURSIVE SEQUENCE.

A **RECURSIVE SEQUENCE** is an ordered list of numbers defined by a starting value and a RULE.

You generate the sequence by applying the rule to the starting value, then applying it to the resulting value, and repeating the process.

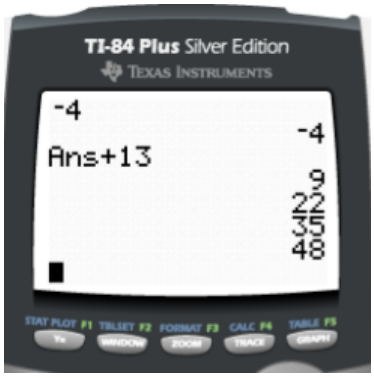
Floor number	Height (ft.)
0	-4
1	9
2	22
3	35
4	
5	
6	
7	
8	
9	
10	
25	

The table shows heights above and below ground at different floor levels in a 25-story building.

Copy this table into your Interactive notebook.

The starting value is -4 because the basement is 4 feet below ground level. Each floor is 13 feet higher than the floor below it, so the **rule** for finding the next floor height is "**add 13 to the current floor height**"

The calculator screen shows how to enter this recursive routine into your calculator.



- **Press -4 --> ENTER to start your number sequence.**
- **Press +13 --> ENTER. The calculator automatically displays ANS + 13 and computes the next value.**
- **Press ENTER again. The calculator applies the rule for finding successive floor heights.**

How high up is the 10th floor?

Count the number of times that you press ENTER until you reach 10.

Which floor is at a height of 217 feet?

How high up is the 25th floor?

Toothpick investigation

Step 1:

Make Figures 1 - 3 of the pattern.

How many toothpicks does it take to reproduce each figure?

How many toothpicks lie on the PERIMETER of each figure?

Figure 1 Figure 2 Figure 3

Step 2:

Make Figures 4–6 from toothpicks by adding triangles in a row. Record the results in the table.

	Number of toothpicks	Perimeter
Figure 1		
Figure 2		
Figure 3		
Figure 4		
Figure 5		
Figure 6		
Figure 10		
Figure 25		

Step 3:

What is the rule for finding the number of toothpicks in each figure? What is the rule for finding the perimeter? Use your calculator to create recursive routines for these rules. Check that these routines generate the numbers in your table.

Step 4:

Now make Figure 10 from toothpicks. Count the number of toothpicks and find the perimeter. Does your calculator routine give the same answers? Find the number of toothpicks and the perimeter for Figure 25. Add those results to the table.

Next you'll see what sequences you can generate with a new pattern.

Step 5 Design a pattern using a row of squares, instead of triangles, with your toothpicks. Repeat Steps 1–4 and answer all the questions with the new design. Record the results in the table.

	Number of toothpicks	Perimeter
Figure 1		
Figure 2		
Figure 3		
Figure 4		
Figure 5		
Figure 6		
Figure 10		
Figure 25		

Step 6:

Choose a unit of measurement and explain how to calculate the area of a square made from toothpicks. How does your choice of unit affect calculations for the areas of each figure?

Now you'll create your own puzzle piece from toothpicks. Add identical pieces in one direction to make the succeeding figures of your design.

Step 7 Draw Figures 1–3 on your paper. Write recursive routines to generate number sequences for the number of toothpicks, perimeter, and area of each of six figures. Record these numbers in the table. Find the values for a figure made of ten puzzle pieces.

	Number of toothpicks	Perimeter	Area
Figure 1			
Figure 2			
Figure 3			
Figure 4			
Figure 5			
Figure 6			
Figure 10			

Step 8 Write three questions about your pattern that require recursive sequences to answer. For example: What is the perimeter if the area is 20? Test your questions on your classmates.

In the investigation you wrote number sequences in table columns. Remember that you can also display sequences as a list of numbers like this:

1, 3, 5, 7, . . .

Each number in the sequence is called a **term.** The three periods indicate that the numbers continue.

Find the missing values in each sequence.

a. 7, 12, 17, __ , 27, __ , __ , 42, __ , 52

b. 5, 1, – 3, __ , – 11, – 15, __ , __ , – 27, __

c. –7, __ , – 29, __ , – 51, – 62, __ , – 84, __

d. 2, – 4, 8, – 16, 32, __ , 128, – 256, __ , __

How many hidden numbers can you find?

For each sequence, identify the starting value and the operation that must be performed to get the next term.

a. The starting value is 7 and you add 5 each time to get the next number. The missing numbers are shown in red.

starting value

+5 +5 +5 +5 +5 +5 +5 +5 +5

7, 12, 17, 22, 27, 32, 37, 42, 47, 52

b. The starting value is 5 and you subtract 4 each time to get the next number. The missing numbers are shown in red.

starting value

−4 −4 −4 −4 −4 −4 −4 −4 −4

5, 1, −3, −7, −11, −15, −19, −23, −27, −31

c. The starting value is −7. The difference between the fifth and sixth terms shows that you subtract 11 each time.

starting value

$$\underset{-7,}{\overset{\downarrow}{}}\ \overset{-11}{\curvearrowright}\ \underset{-18,}{}\ \overset{-11}{\curvearrowright}\ \underset{-29,}{}\ \overset{-11}{\curvearrowright}\ \underset{-40,}{}\ \overset{-11}{\curvearrowright}\ \underset{-51,}{}\ \overset{-11}{\curvearrowright}\ \underset{-62,}{}\ \overset{-11}{\curvearrowright}\ \underset{-73,}{}\ \overset{-11}{\curvearrowright}\ \underset{-84,}{}\ \overset{-11}{\curvearrowright}\ -95$$

d. Adding or subtracting numbers does not generate this sequence. Notice that the numbers double each time. Also, they switch between positive and negative signs. So the rule is to multiply by −2. Multiply 32 by −2 to get the first missing value of −64. The last missing values are 512 and −1024.

starting value

$$\underset{2,}{\overset{\downarrow}{}}\ \overset{\cdot(-2)}{\curvearrowright}\ \underset{-4,}{}\ \overset{\cdot(-2)}{\curvearrowright}\ \underset{8,}{}\ \overset{\cdot(-2)}{\curvearrowright}\ \underset{-16,}{}\ \overset{\cdot(-2)}{\curvearrowright}\ \underset{32,}{}\ \overset{\cdot(-2)}{\curvearrowright}\ \underset{-64,}{}\ \overset{\cdot(-2)}{\curvearrowright}\ \underset{128,}{}\ \overset{\cdot(-2)}{\curvearrowright}\ \underset{-256,}{}\ \overset{\cdot(-2)}{\curvearrowright}\ \underset{512,}{}\ \overset{\cdot(-2)}{\curvearrowright}\ -1024$$

APPLICATION In the Empire State Building the longest elevator shaft reaches the 86th floor, 1050 ft above ground level. Another elevator takes visitors from the 86th floor to the observation area on the 102nd floor, 1224 ft above ground level.

a. Write a recursive routine that gives the height above ground level for each of the first 86 floors. Tell what the starting value and the rule mean in terms of the building.

APPLICATION In the Empire State Building the longest elevator shaft reaches the 86th floor, 1050 ft above ground level. Another elevator takes visitors from the 86th floor to the observation area on the 102nd floor, 1224 ft above ground level.

b. Write a recursive routine that gives the heights of floors 86 through 102. Tell what the starting value and the rule mean in this routine.

APPLICATION In the Empire State Building the longest elevator shaft reaches the 86th floor, 1050 ft above ground level. Another elevator takes visitors from the 86th floor to the observation area on the 102nd floor, 1224 ft above ground level.

c. When you are 531 ft above ground level, what floor are you on?

d. When you are on the 90th floor, how high up are you? When you are 1137 ft above ground level, what floor are you on?

Summary: I might use a recursive sequence in real life when I...

Out: Explain the calculator steps need to find the 24th number in a recursive sequence that has a starting value of -20 and adds 17 to each new value.